Load Dispatch Management Using Trend Analysis of Demand and Generation in Pakistan

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Abstract-Load management has been an enigma despite the adequate level of production of electricity. This research focuses on the short-term evaluation of the maximum output of various electricity generation facilities to meet the required system load at minimum possible cost without disrupting the regular power supply to the end users and aimed to give overview of the economic load dispatch problem regarding abrupt variations in monthly load demand and generation throughout the year. The research analysis was conducted by using daily data regarding the generation, load demand across the country and power system constraints of generation and transmission. The aspect of reduction in share of Hydel generation during canals closure at water scarcity season every year has also been taken into account. By analyzing both the actual and predicted values at load dispatch center, it provides monthly trend of generation, consumer demand and load shed at the same time. Daily and monthly data figures of the last 06 years in MW/GWh is analyzed and executed. Values have been plotted in graphs to understand the monthly/annual variations in total generation and total demand. Research is aimed to improving the quality of power dispatch and deriving workable solutions.

Keywords-Load Dispatch Management (LDM), Economic Dispatch(ED), Demand Side Management (DSM), Unit Commitment (UC), Mega Watts(MW), Giga Watt Hours (GWh).

I. INTRODUCTION

Pakistan is one of the countries that is seriously facing energy crisis due to increase in energy demand as of globally. Though there are many factors but the most common include increase in population size, luxurious standards of living, and reduction in availability of energy resources in developing countries [i]. Some future assumptions concluded from researches, show that by year 2040 the overall population of world may increase from 7 billion to 9 billion due to which energy demand can be increased up to 35% than current requirement [ii-iii]. There are many causative factors for Residential energy usage like increasing population, increasing urbanization and upgraded living standards [iv-vii]. The installed electricity generation capacity of Pakistan was 22,797 MW in 2013 whereas the average demand was 17,000 MW and hence, the shortfall was between 4,000 and 5,000 MW. The percentage share of sources of generation was as about Oil (35.2 per cent), Hydel (29.9 per cent), Gas (29 per cent) Nuclear, solar and imported (6 per cent). As per our analysis, the electricity demand during peak hours will rise by 4 to 5 percent in the next 10 years, roughly 1,500 MW. This depressing forecast is the product of a lopsided energy mix, fading indigenous fuel reserves, expanding circular debt and transmission hold-ups. At the current stage, economic growth depends on the availability and affordability of sustained energy supplies and vice versa [viii].In Pakistan, the energy demand is also increasing exponentially due to a rapidly growing population and the number of households. Pakistan stands at six amongst the most populous countries in the world, having a population over 180 million in 2013, and the future estimates show over 350 million citizens in 2035 [ix].The gas reserves in Pakistan are close to exhaustion. The price hike in imported oil affect the budget considerably and therefore, it is not practically possible to ensure its constant supply. Hydel power seems to be the only probably solution to meet the upcoming energy challenges but the inherent and obvious political and environmental issues make building dams a farfetched opportunity. In the current scenario, perhaps, reducing excessive reliance upon oil and adopting environment friendly and low-cost alternatives are the best options. Presently in Pakistan there are severe energy crises having devastating effects on economic growth [x]. This results in increasing unemployment and mental stress. Activities related to public or community welfare services have also met a decline.

Maintaining a sound balance between demand and generation is certainly an integral operational function in the modern clay energy management system. The reason behind use of the LD is to identify the maximum generation within the existing operating units and identification of ways through which the total generation cost gets minimized without affecting the power balance equation and eradicating the other constraints affecting power supply in the system. During analysis of the existing literature on the ED issue and probable solutions it is realized that the traditional techniques and strategies to counter the issues prove to be excessively complicated while dealing with complex dispatch problems. Also evident was the fact that the lack of robustness and efficiency was prevalent in numerous practical applications. An Efficient and optimum electric load dispatch in centralized power supply systems has always held an important position in the electric power industry. Over the past few decades, it has become highly important to ensure that the power systems function face minimum interruptions and consistent power supply is maintained to satisfy consumers throughout the year. Keeping the generation system updated and in optimal condition are also important concerns. Considering the small number of efficient generating units in terms of production costs and increasing power supply demand, fuel cost and supply limitation, it is important that the committed units generate the expected load. Also, it is vital that the expected load generation remains undeterred by fluctuations in fuel cost and varying load demand at various time intervals [xi-xiv].

The primary objective of LD of electric power generation is to program the committed generating unit outputs in order to meet the load demand while utilizing minimal operating cost as well as satisfying all equality and inequality constraints in units and system. The LD issue has been addressed by different researchers already and the issue is an extensive one involving different problems. Some of the main problems include the Unit Commitment or Pre-Dispatch issue, which involves selection of most efficient generating sources to meet the expected load demand and ensure a specified margin of operating reserve within specific time duration. Another problem is the on-line committed dispatch of load, which involves extended distribution to the end users. In centralized LD, the electricity supply is not fixed because it is received via different generation units. However, the units are allowed to take values again with specific limits to fulfill a certain load demand with minimal fuel consumption [xv-xvi].

Load Management is basically the process of balancing electrical load with the electricity supply on the network by adjusting or controlling the load instead of controlling the power station output, which is also referred to as demand side management (DSM). This is achieved by real-time direct interference of the utility using frequency sensitive relays triggering the circuit breakers (ripple control) via time clocks or alternately, by utilizing especial tariffs to modify consumer behavior. Through Load Management, utilities can minimize electricity demand during peak hours and this resultantly, reduces costs by eradicating the need for tweaking power plants. Additionally, power plant optimization is a time-consuming process as the requirement of bringing them on-line and addressing of challenges in case a plant goes off-line unexpectedly. Load management also helps in reducing harmful emissions because peaking plants or backup generators are less efficient, less environment-friendly and unreliable options than base load power plants. Latest load-management technologies are being developed and modified continually by both private and public sectors.

Real-time dispatch can be computed in two stages. The first stage involves solving of a unit commitment (UC) problem for selecting generating units in order to meet the required per hour load demand [xvii-xviii]. The second stage involves solving of an ELD problem for computing the power outputs of the committed units for meeting the required load. This decision is usually taken minutes to hours before the implementation time. Current studies on UC have focused on planning purposes to accommodate load and generation forecasting challenges.

National load dispatch center is the department responsible for controlling and monitoring of the country's power supply mechanism. The primary tasks include the Dispatching power from available sources of generation and Consistent processing of the power transmission network. This center is based at Islamabad and is required to conduct generation, dispatch and operation of 500/220 kV primary transmission network. On the other hand, is responsible for maintaining the quality of power in terms of frequency and voltages.

Dispatching of electricity is indeed a dynamic task and control center is responsible for performing this daunting task along with addressing the technical, seasonal, financial and social aspects that affect power dispatch. The department controls electricity generation through hydro-thermal coordination considering the water releases from the dams as sanctioned by IRSA. It could be affected by anything, a public holiday, a Pak-India cricket match, abrupt and uncalled for weather changes or sudden changes in power demand during the month of Ramadan. Load dispatch management center faces a new challenge every day in maintaining consistent power supply and meeting regular demand. Apart from this, the center is responsible for conducting studies on power system to evaluate load flow and contingency analysis, load forecasting and short to medium term operational planning. It also has to examine, analyze and schedule the maintenance shutdowns of generation units and transmission equipment as well as ensuring system's stability and security for uninterrupted load management.

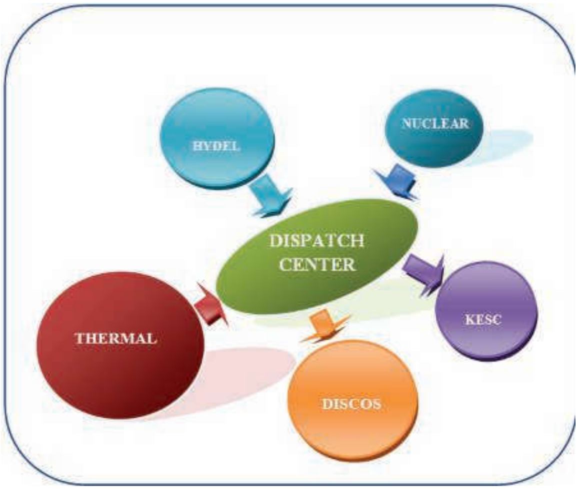

Fig. 1. Power System Structure of Dispatch Center

The two fundamental components of load dispatch are described below:

A. Planning for Tomorrow's Dispatch

- Proper scheduling of generation units for every hour of the next day's dispatch according to the forecast load.
- Selection of most appropriate available generating units for operation to dispatch load for the next day (operating day).
- Recognizing capacity of each generating unit.
- Ramping rate (The response time of any generator in case it is put in operation)
- Recognizing maximum and minimum generation levels of each Generating unit.
- Recognizing value of minimum time limit the generating unit must stay off when once turned off.

2. Dispatching for the Power System Today

- By using Automatic Generation Control (AGC) for the purpose to change generation dispatch as needed, monitor and maintain system frequency at 60 Hz
- Monitor each hour dispatch plans for ensuring balanced dispatch in every hour
- Observing transmission system flows
- Ensuring that transmission flows remain in reliable limits
- Ensuring that voltage levels are within reliability ranges
- Controlling new power flow timetables
- Restraining current power flow timetables
- Altering the dispatch
- Shedding Load.

In electrical power sector, the Load Dispatch mechanism is a critically important job because it involves ensuring balanced and uninterrupted power supply to the end users within minimal cost. Since the efficiency of the new generating units is higher than the older ones, therefore, the economic dispatch has to be resolved efficiently and tactfully to reduce the power generation cost. Load dispatch problems are here analyzed in last 06 years data of generation, demand and load shed. Maximum computed demands in a month and corresponding capabilities of Private Power Plants, Thermal and Hydel sources are analyzed, which results in a good idea of load management with respect to generating capability of plants connected to WAPDA system and consumer demand throughout the year. Conclusions are arrived which finally leads to an outline of the future directions for research and development efforts in this particular area.

II. METHODOLOGY

However, in Pakistan, the higher order cost functions for (a) improved curve fitting for running cost, (b) lower estimation, (c) much practical, precise and reliable results, regression analysis for last 06 years is done for load dispatch center. In this paper numerical data from NTDC and WAPDA Power Plants (64), Generating Units (496), 500 kV Stations (17), 220 kV Stations (35), 132 / 66 / 33 kV Station (900), 500 kV Transmission Lines (5183) KM, 220 kV Transmission Lines (9104) KM, 132 / 66 / 33 kV Transmission lines, (28,892) KM is used. We analyzed and compare the period of mismatch between demand and generation throughout the year, uncertainty of availability of electricity generated by Hydel, thermal and nuclear from both generation and demand perspective over a variety of times scales. How load dispatch management mainly keeps balance between total generation and total power demand, the difference is named as load shedding or load management. So six years data is analyzed on daily and monthly basis for better regression analysis. The general form of the estimated regression equation for modeling the linear trend in the energy generation is $\hat{Y}_t = b0 + bY$ also satisfied. Where, $\hat{Y}_t$ is forecast of energy generation in period t and Y is the number of year i.e. 1 for 2009, 2 for 2010 and so on. Also multiple regression equation for modeling both the month wise seasonal effects and the linear trend in the energy generation time series is discussed in equation, $\hat{Y}_t = b0 + b1M1 + b2M2 + \dots \dots \dots + b12M12 + b13t$. Where $\hat{Y}_t$ is forecast of energy generation in period t and M1 …. M12 are the number of months i.e. 1 for January and 12 for December is taken, if period t correspond to January then 1 otherwise 0 and so on for each month. Time period t i.e. considering time in fiscal years July 2009 is 1 and June 2015 is taken as 72.In results the equations clearly shows the difference between actual and predicted values of demand, generation and load shed by the load dispatch center.

III. DATA TREND ANALYSIS AND DISCUSSIONS

All the data daily, monthly and annual is taken and analyzed, the results taken are as under:

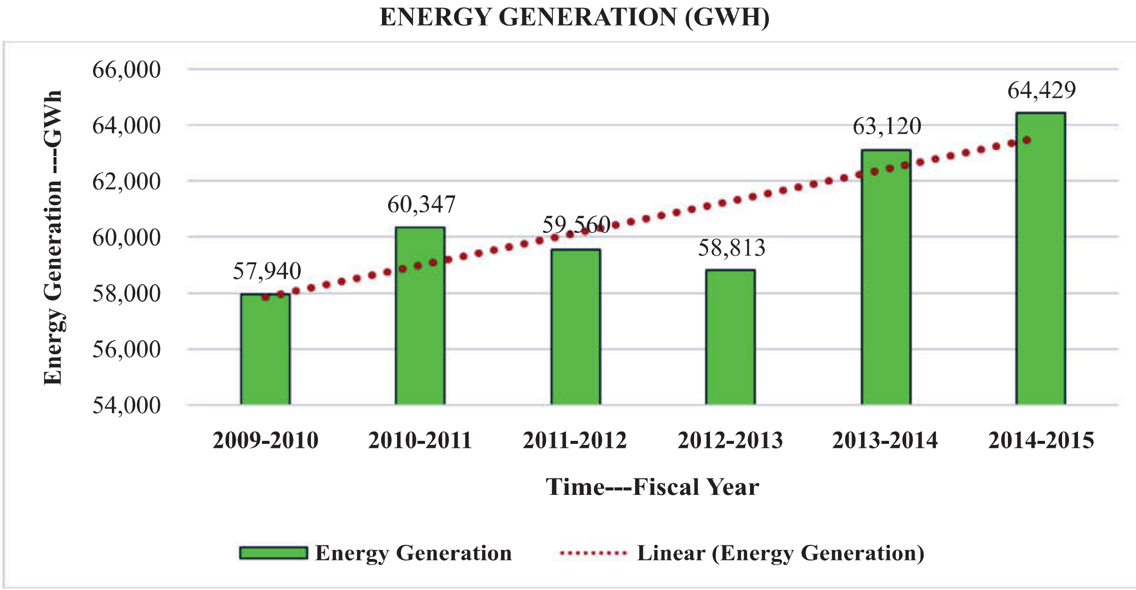

Fig. 2. Actual annual Growth in Net Energy Generation

The general form of the estimated regression equation for modeling the linear trend in the energy generation is as follows:

$$Y \quad t = b0 + bY$$

Where

Y t̂ is forecast of energy generation in period t and Y is year number i.e. 1 for 2009, 2 for 2010 and so on.

In Fig.2,the mean of above data is 60,701GWh having range of 6,489GWh from 57,939GWh to 64,428GWh. The data covers six years from July 2009 to June 2015. Standard deviation of the data is 2,544 GWh and data is slightly skewed positively to right by 0.666. ??? Slope of the line is 1,143.4, whereas intercept is 56,700. Hence the linear equation is y = 56,700 + 1,143.4 (Y). . R2 of the data is 0.7068, which reflect that data is quite closer to the trend line. This aspect is also clearly evident from the Energy Generation Graph in Fig. 2.

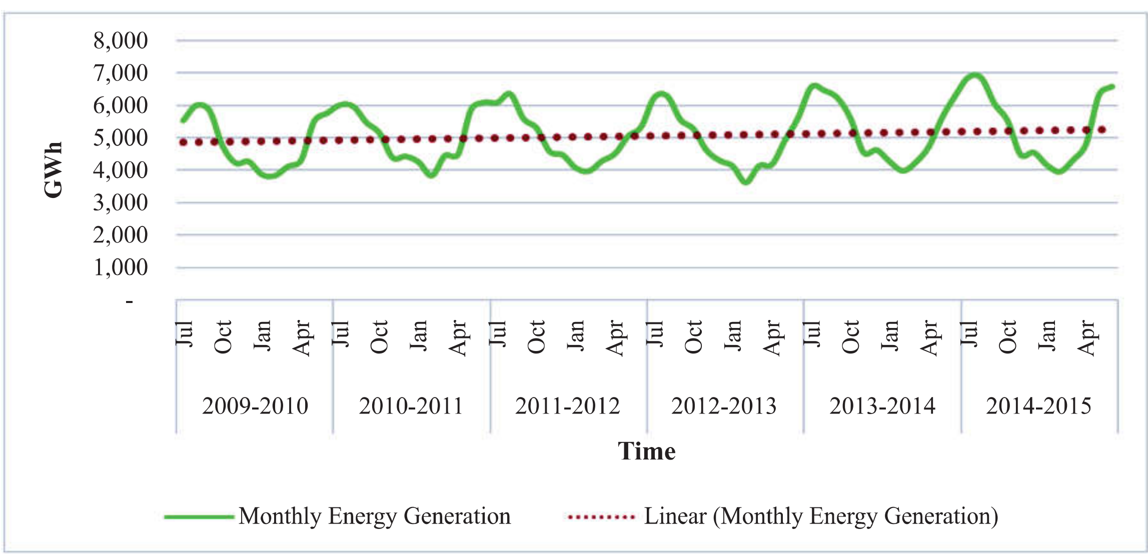

Fig. 3. Monthly Energy Generation

In Fig. 3, We have observed a clear pattern of movement in the monthly energy generation chart during the each six months of a year. That is, from February to July, the energy generation increases and declines linearly from August to January. Slope of the data is 5.21 whereas intercept is at 4867.14. R2 of the data is 0.015, which reflects large variation of data around trend line and it is also clear from monthly energy generation chart in Fig.3 that data crossed the trend twice in a year.

The general form of the estimated regression equation for modeling the linear trend in the energy generation is as follows:

$$Y \quad t = b0 + bM$$

Where

$Y \quad t$ is forecast of energy generation in period t and M is month number i.e. 1 for July 2009 and 72 for June 2015.

The linear equation concluded from this data is $y = 4867.14 + 5.21$ (Month)

In order to further investigate the pattern of energy generation and to confirm the understanding that there is a seasonal effect and linear trend, regression is applied on the data. Then the estimated multiple regression equation for modeling both the monthly seasonal effects and the linear trend in the energy generation time series is as follows:

Yt̂= b0+ b1M1 + b2M2 + ………. + b12M12 + b13t

Where

Yt̂ is forecast of energy generation in period t and M1 …. M12 are month numbers i.e. 1 for January and 12 for December, if period t corresponds to January then 1 otherwise 0 and so on for each month

T is time period i.e. July 2009 is 1 and June 2015 is 72

Based on the regression model using dummy variable, following equation has been drawn to compute energy generation for any month in a year. Results of the regression model are reasonably good. R2 of the data is 0.93, which shows the strength of linear equation:-

y = 5613.64 + 359.15 Jul + 450.62 Aug - 86.48 Sep – 623.36 Oct – 1431.23 Nov – 1469.21 Dec – 1796.59 Jan – 2061.80 Feb – 1675.79 Mar – 1435.90 Apr – 388.87 May – 0 Jun + 7.96 Period

In Fig. 4. below are the graphical results of the equation concluded above. It is evidently apparent that

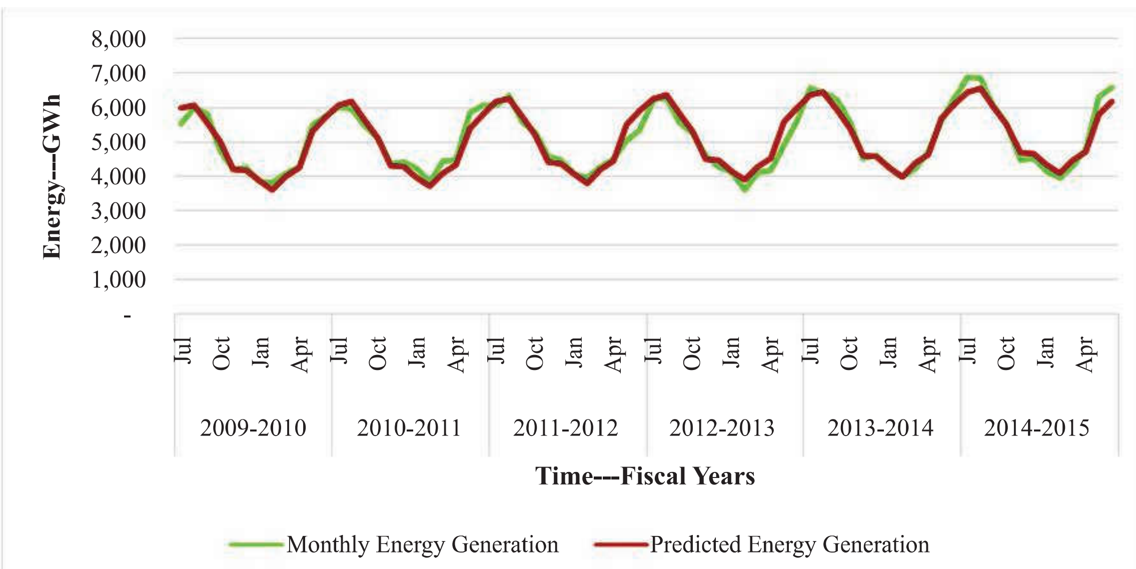

Fig. 4. Monthly Energy Generation Actual vs. Predicted

IV. LOAD DEMAND

Tremendous increment in electricity demand has been dubbed as the most prominent factor contributing to the obstinate demand-supply gaps. This has been proven by various studies conducted on the issue of power crisis involving aspects like governance, transmission, distribution losses and circular debt etc. During the past three decades there has been enormous upsurge in the electricity demand most probably due to factors like industrialization, urbanization, rural electrification, agriculture and service sector growth, increasing domestic demand and rising per capita income, actual demand was never fully anticipated due to the lackluster forecast and future planning. It is important to upgrade old plants and to set up new generating stations in the face of constantly increasing electricity demands.

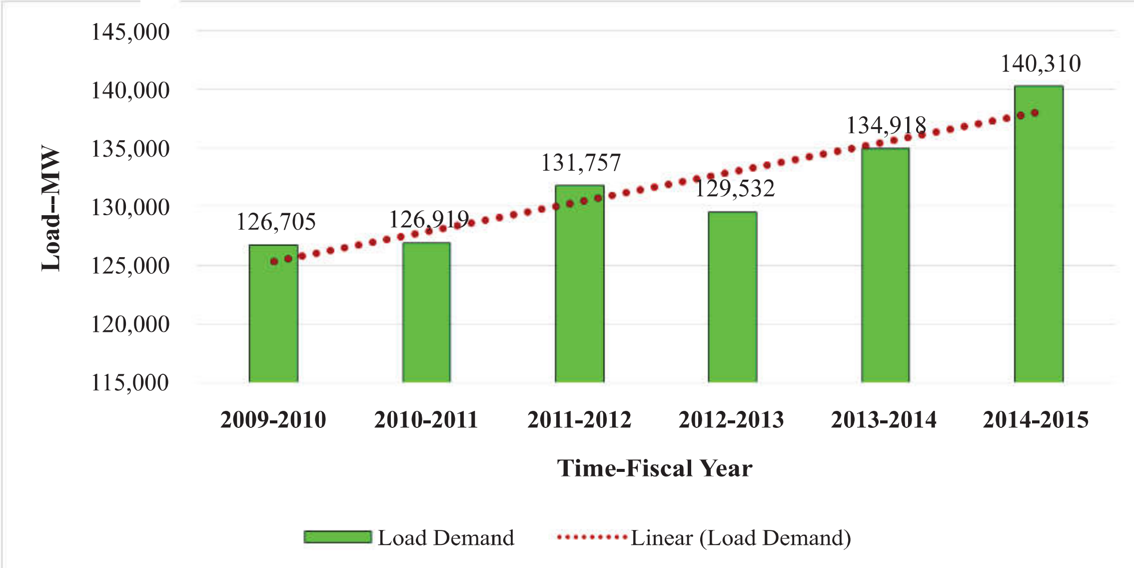

Fig. 5. Yearly Load Demand by Consumers

Six year's period from Jul 2009 to Jun 2015 has been used to analyze load demand. Mean of load demand is 131,690 MW, whereas range of data is 126,705 MW to 140,310 MW(13,605). Standard deviation of the data is 5,234 MW and skewness of the data is slightly positive i.e. 0.9, which shows growing trend.

The general form of the estimated regression equation for modeling the linear trend in the energy demand is as follows:

$$Y \quad t = b0 + bY$$

Where

Yt̂ is forecast of energy demand in period t and Y is the year number i.e. 1 for 2009, 2 for 2010 and so on.

Slope of the annual demand is 2565.54 MW, whereas intercept is at 122,710 MW. R2 of the data is 0.84, which shows the strength of linear equation. Following linear equation has been calculated to predict annual load demand:

$Y = 122{,}710 + 2565.54\ (x\ Year)$

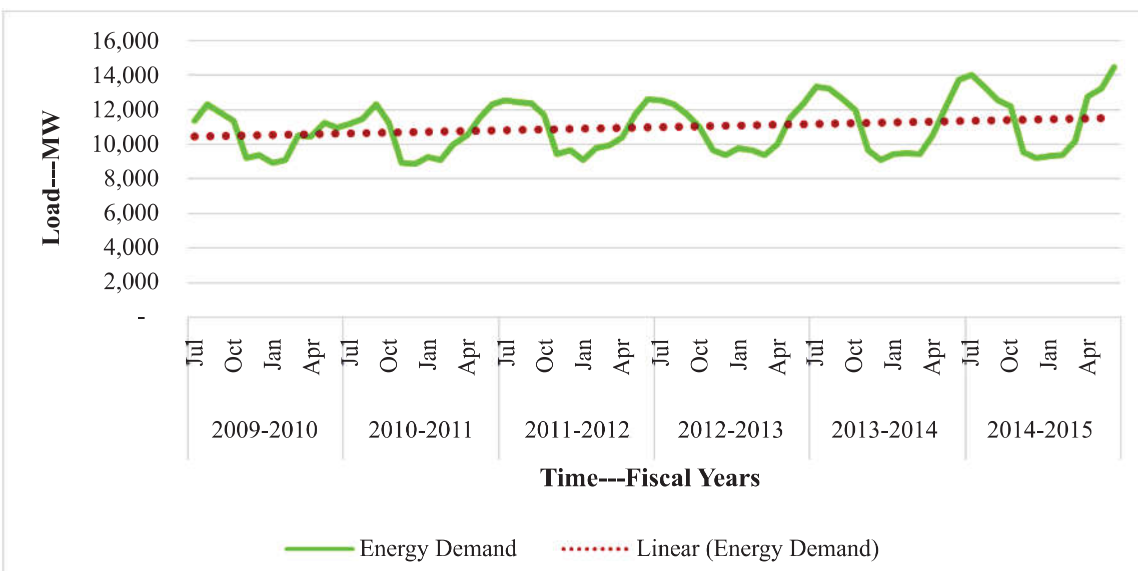

Fig. 6. Monthly Trend of Load Demand By Consumers

The monthly data of energy demand has been analyzed to observe the trend in demand and the graphical results revealed that there is a linear trend in six years. An interesting point to be discussed is the seasonal effects in energy demand, which is also clear from the monthly energy demand chart.

The general form of the estimated regression equation for modeling the linear trend in the energy demand is as follows:

$$Y \quad t = b0 + bM$$

Where

Y t̂ is forecast of energy demand in period t and M is month number i.e. 1 for July 2009 and 72 for June 2015.In fig.6 the Slope of the data is 15.26, intercept is 10417.33 so the best fit line equation is:-

$$y = 10417.33 + 15.26\ (x\ month)$$

However, the R2 of the data is 0.044, which reflects variation of data around trend line and it is also clear from monthly energy demand chart that data crossed the trend twice in a year. In order to conclude strong results and seasonal impacts, data has been further analyzed.

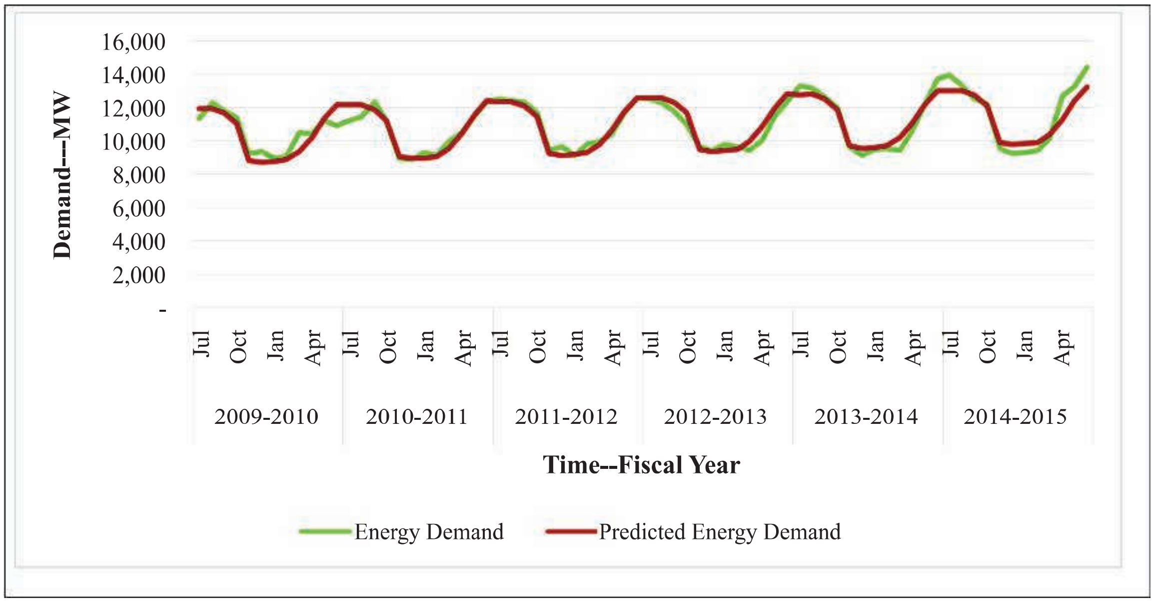

Fig. 7. Variations in actual and predicted monthly demand

The data has been regressed to analyze the seasonal impacts and the results are remarkably positive. After incorporating the seasonal impacts, the predicted values are very close to the actual values, which is clear from above Fig. 7.

The general form of the estimated multiple regression equation for modeling both the monthly seasonal effects and the linear trend in the energy demand time series is as follows:

$$Y \quad t = b0 + b1M1 + b2M2 + \dots \dots \dots . . + b12M12 + b13t$$

Where

Y t̂ is forecast of energy demand in period t and M1 …. M12 are number of months i.e. 1 for January and 12 for December, if period t corresponds to January then 1 otherwise 0 and so on for each month, Time period T, i.e. July 2009 is 1 and June 2015 is 72. R2 of the data is 0.88, which shows the strength of linear equation given below:

$$\begin{aligned} Y = 12000.57 &+ 17.81\,Period - 43.31\,Jul \\ &- 51.63\,Aug - 337.51\,Sep \\ &- 1010.22\,Oct \\ &- 3207.53\,Nov \\ &- 3359.13\,Dec - 3331.8\,Jan \\ &- 3244.58\,Feb - 2783.44\,Mar \\ &- 1932.94\,Apr - 818.01\,May \\ &+ 0\,Jun. \end{aligned}$$

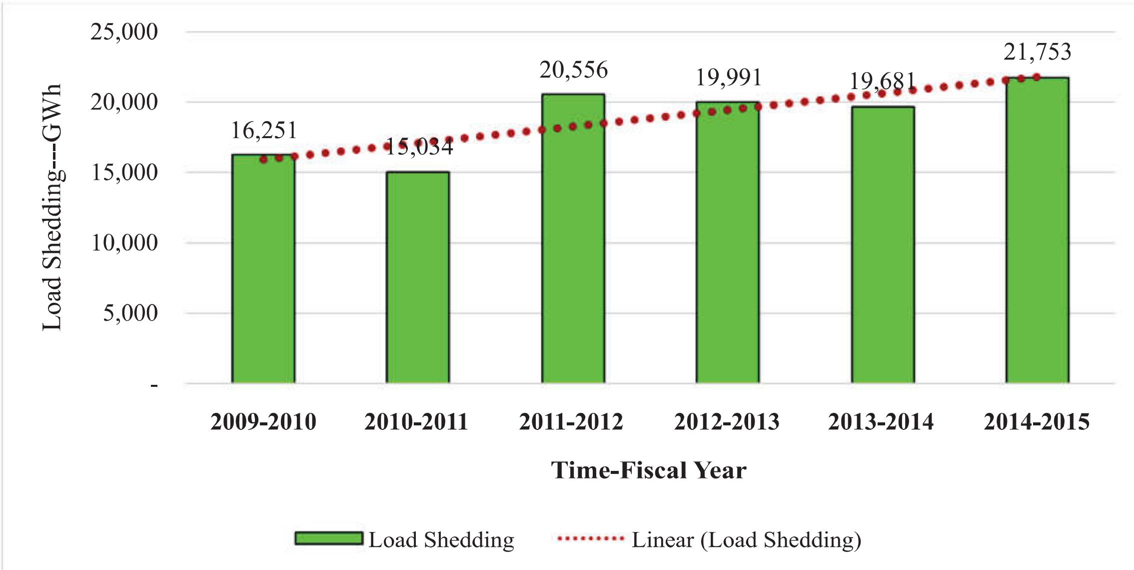

Fig. 8. Load Shedding (Annual Trend)

The range of the data is from 15,034 Gwh to 21,753GWh (i.e. 6,719), mean is 18,878 Gwh and standard deviation of the data is 2,632GWh. This reflects the variation of data around mean.

The general form of the estimated regression equation for modeling the linear trend in the energy load shedding is as follows:

$$Y \quad t = b0 + bY$$

Where

Yˆt is forecast of energy load shedding in period t and Y is year number i.e. 1 for 2009, 2 for 2010 and so on.

In Fig. 8, Slope of the annual energy load shedding is 2,565, intercept is 122,710GWh and R2 of the data is 0.84. The strength of linear equation is quite well. The equation computed is y = 2,565 + 122,710 (x year).

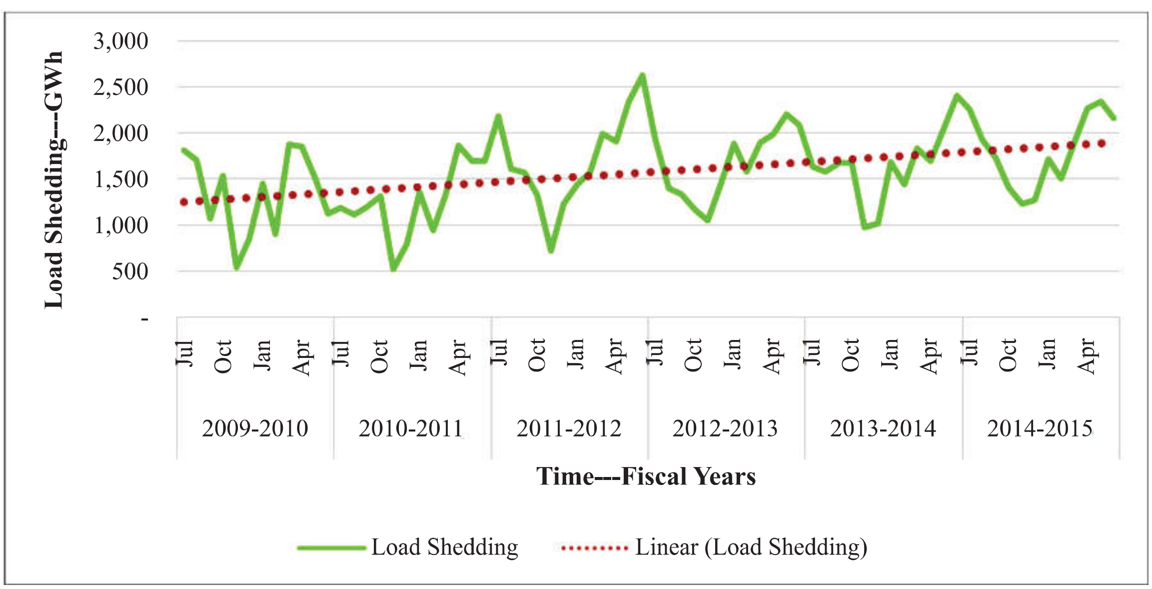

Fig. 9. Monthly Variations in Actual Load Shedding

In order to gain deeper insight of load shedding, monthly load shedding data has been analyzed to assess the trends and seasonal variation. In fig. 9, it is clear that Load shedding data also has seasonality effects. The general form of the estimated regression equation for modeling the linear trend in the energy load shedding is as follows:

$$\hat{Y}t = b0 + bM$$

Where

Yt̂ is forecast of load shedding in period t and M is month number i.e. 1 for July 2009 and 72 for June 2015. The equation drawn for monthly load shedding is given below. However, R2 of the data is 0.17, which is quite low and reflect the variation of data around the linear line:-

$$Y = 9.2 + 1{,}237\ (x\ month)$$

To analyze the effects of seasons the data is being further analyzed below.

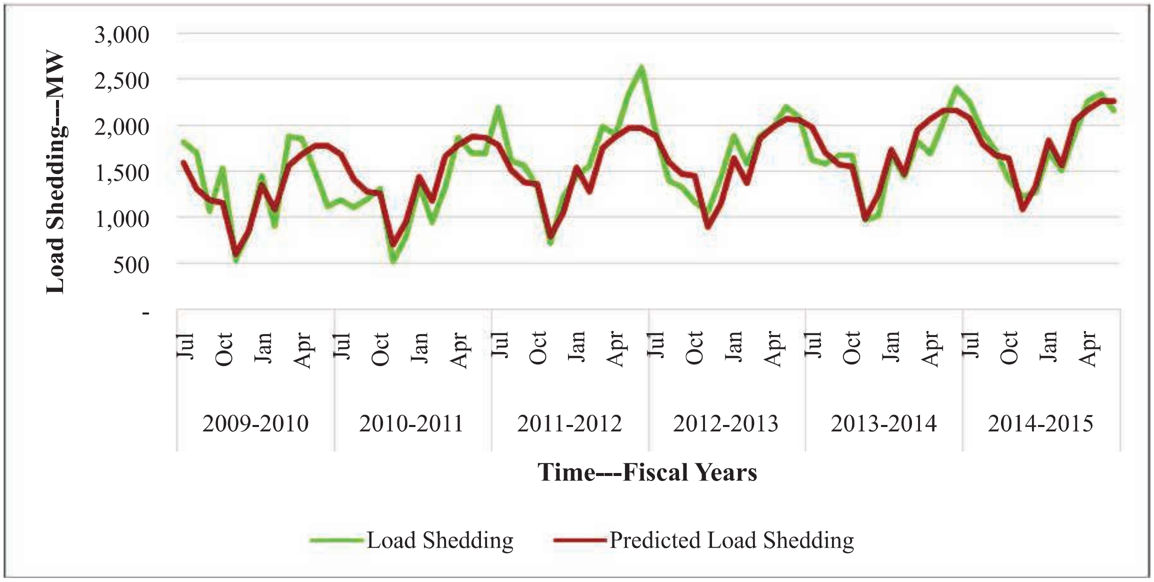

Fig. 10. Monthly Load Shedding Trend – Actual Vs Predicted

The data has been regressed to analyze the seasonal impacts, after incorporating the impacts, the predicted values are very close to the actual values, which is clear from above figure 10.The general form of the estimated multiple regression equation for modeling both the monthly seasonal effects and the linear trend in the energy load shedding time series is as follows:

$$Y\quad t = b0 + b1M1 + b2M2 + \dots\dots\dots.. + b12M12 + b13t$$

Where

Yt̂ is forecast of energy load shedding in period t and M1 …. M12 are the month numbers i.e. 1 for January and 12 for December, if period t correspond to January then 1 otherwise 0 and so on for each month, Time period T, i.e. July 2009 is 1 and June 2015 is 72. R2 of the data is 0.75, which shows the strength of linear equation given below:-

$$Y = 1678.77 + 8.11\,Period - 91.96\,Jul - 379.72\,Aug - 516.61\,Sep - 547.97\,Oct - 1120.24\,Nov - 870.39\,Dec - 385.86\,Jan - 660.65\,Feb - 187.22\,Mar - 72.59\,Apr + 12.26\,May + 0\,Jun.$$

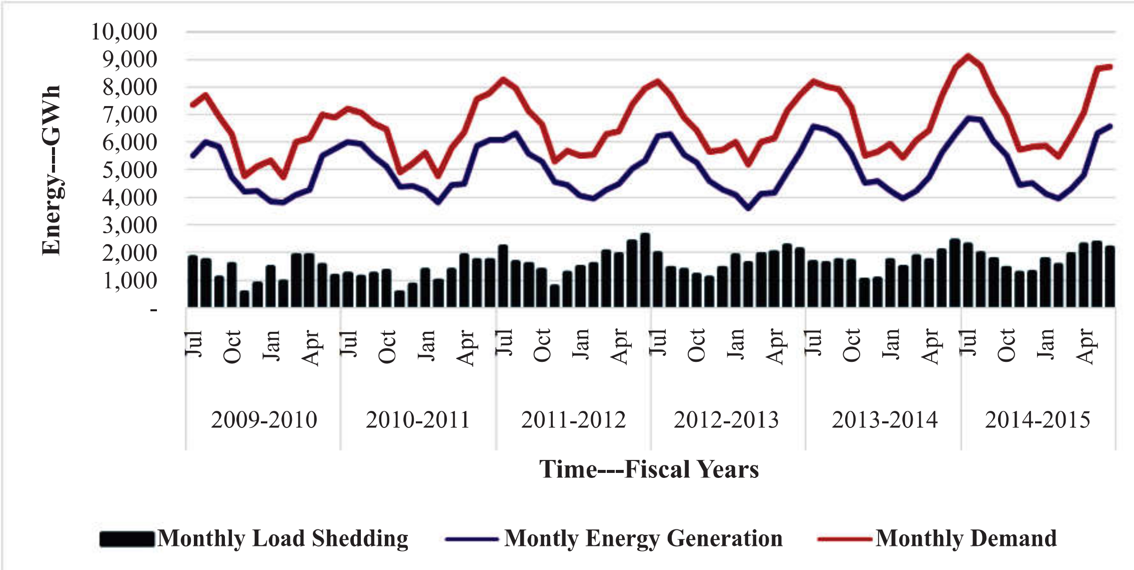

Fig. 11. Comparison of Energy Generation, Demand and Load Shedding

V. Conclusion

The analyzed research data of a period of 6 years from WAPDA's own power plants generation, Private Power plants and purchased from KESC has shown a significant room of improvement in load dispatch system. Different Load management measures were implemented in last 6 years to address the load management issues with different constraints and certain power system impediments. The major impediments steered with the whole load management systems were, less hydel generation due to canal closure in January of each year, sudden forced outage of WAPDA, Thermal and IPP's, transformation due to scheduled and emergency maintenance of power plants. The permanent impediment is the less hydel generating potential due to lower reservoir level with fewer outflows to meet the water indents allocated by IRSA and occasional transmission/transformation bottlenecks that further provokes for better load management opportunity. Results show generation increases in 4 years whereas decreases in 2 years. Each of the resulting values corresponds to previous year. The results on demand side show unsteady growth as 7.89%, 0.16 %, 3.87 %, 3.72 % and 3.93 % rise in years, 2009-2010, 2010-2011, 2011-2012, and 2013-2014 respectively, whereas, a decrease at the rate of 1.53% during the year 2012-2013.This research will be a future guideline for economic dispatch centre to address the concurrent and future issues with comprehensive load management planning. For future, the solar and windmill generation will be a significant contributor in energy generation after net metering and should be considered for future research.

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